It Takes Time to Measure Time

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Abstract

Measurement of time is a process that takes time. This creates a lag between the measurement and the flow of time. The lag emerges from the definition of time and is not due to any physical limitation. Perfect measurement of time, therefore, may not be logically possible.

What is a Clock?

A clock is a machine that measures time. It requires two essential components [2]:

1. A regular process component. This component ideally represents the orderly and consistent flow of time, like an ideal pendulum in a pendulum clock or a photon in a light clock.

2. A measurement process component. This component measures the flow of time of the first component.

One might wonder, how to know if a process is indeed regular? We need a clock to decide. But to build a clock, we need to have a regular process! This circularity is unavoidable, and thus a "Perfect Clock" is logically not feasible [1]. However, for the sake of argument, let us assume that we are able to identify a perfectly regular process.

Time as Computation

To avoid issues of physical limitations, let us assume that each component of the clock is a Turing Machine, i.e., an ideal computer machine. Such ideal computers have no physical limitations; they have zero friction and an unlimited supply of energy and memory [4].

Let us denote the first component as TM_1 while the second as TM_2. The regular computations of TM_1 represent the flow of time, and the output of each regular computation is passed on to TM_2, based on which the latter computes the current date and time. The two machines are assumed to be of equivalent efficiency and power.

There is a rich discussion on how to measure the time of computer programs ([4], chapter 12). For simplicity, we assume that each computational step of TM_1 measures a unit of time flow.

While the computations of TM_1 are regular, the measurement computations by TM_2, in general, need not be regular. Moreover, time measurement is subject to a variety of conditions related to the type of calendar to be adopted, e.g., the number of hours per day, the number of days per month, holidays, etc. This indicates that TM_2 will have more rules (or states) than TM_1, making the potential number of computations for the measurement relatively larger.[1]

What is the Problem?

Here is the problem: The measurement of time by TM_2 is also a computation. When TM_2 completes the measurement, TM_1 will have already progressed in its regular computations. The flow of time will not be suspended until the measurement is completed; rather, *the measurement takes place in time.* But this means that as TM_2 receives the output of the next regular computation of TM_1, a shorter period would have elapsed between the two measurements. Given that the computations of TM_2 are generally subject to a larger number of rules than TM_1, as indicated above, there are good chances that the lag will gradually accumulate till the point when the measurement is no more reliable.

The Prediction Problem

Is it possible for TM_2 to predict the lag in time measurement and adjust its output accordingly?

Suppose it is. This means that TM_2 must perform the following series of computations:

1. Simulate the computations of TM_1.
2. Calculate the current date and time.
3. Calculate the "time" it takes to complete its calculations in 2.
4. Calculate the lag between 1 and 3.
5. Announce the adjusted date and time.

[1]As implied by the Busy Beaver function; see:[4], p. 108.

There are two problems here:

First, time is defined in terms of the regular computations of TM_1. The only way to register the flow of time is through the periodic output of TM_1. There is no obvious formula that determines the number of computations of TM_2 in terms of those of TM_1. Put differently, TM_2 has no internal "clock" to find out the time needed for its computations, or else we end up with another source of circularity because we are now analyzing the internal parts of a clock. Thus, without the periodic output of TM_1, TM_2 cannot calculate the time needed to complete its calculations. Hence, step 3 above is probably not possible.

Second, even if possible, TM_2 needs to complete the computations in 1-5 above exactly as TM_1 completes a unit flow of time, so that the timing of the output of TM_2 will match that of TM_1 to avoid any lag. This means that TM_2 must complete the simulated computations of TM_1 *before* TM_1 does, in order to leave some room for the remaining steps. But if TM_1 and TM_2 are of equivalent power, how could TM_2 execute the same number of computations faster?[2]

Conclusion

The measurement of time takes time. This creates a lag between the flow of time and its measurement. This lag arises from the definition of time and not because of any physical constraints. It is a logical gap that cannot be avoided. This is another reason why a "Perfect Clock" may not be logically possible.

References

[1] Al-Suwailem, S (2022) "The Impossibility of a Perfect Clock," *Authorea*, December 15.

[2] Orzel, C. (2022) *A Brief History of Timekeeping*, BenBella Books.

[3] Popper, K. (1956/1982) *The Open Universe*, Routledge.

[4] Reus, B. (2016) *Limits of Computations*, Springer.

[2]This is very close to the problem of predicting the self-growth of knowledge that Karl Popper ([3], section 22) discusses in detail